Technical Note

On average heat transfer coefficient

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In the most general case of steady state heat transfer from a finite surface such as the wall of a heat exchanger passage, the thermal parameters vary with position. The parameters of interest are the wall and fluid temperatures, T_w and T_b respectively, the normal component of the heat flux vector at the wall, q_w, and, accordingly, the heat transfer coefficient h. The local heat transfer coefficient h_x is defined as:

$$h_x = \left\{ \frac{q_w}{T_w - T_b} \right\} \tag{1}$$

and serves as a yardstick for comparison of the performance of various surfaces. The local values of these parameters are such as to satisfy energy conservation for conjugate heat transfer in the fluid and in the wall. Indeed the wall itself may contain energy sources or sinks, be heated or cooled externally, and be of variable thickness. However, at any point on the fluid–wall interface, there are unique values of q_w, T_w and, hence, h_x, it being tacitly assumed that the local bulk (or stream) temperatures T_b of the fluid is known.

In engineering calculations, the average values for the surface are of interest, and the adoption of average wall temperature $\bar{T}_w$, average heat flux $\bar{q}_w$, and average heat transfer coefficient are commonplace. With regard to temperature and heat flux, there is no ambiguity as to what is intended. The average wall temperature for example along a surface of length L in direction x (see Fig 1(a)) is simply

$$\bar{T}_w = \frac{1}{L} \int_0^L T_w \, dx \tag{2}$$

and the same type of expression applies to the average heat flux. However, confusion sometimes arises with average heat transfer coefficient $\bar{h}$, for there are at least two possible definitions. In the mathematical context 'average' means arithmetic mean, and, accordingly, one definition is

$$\bar{h}_x = \frac{1}{L} \int_0^L \frac{q_w}{T_w - T_b} \, dx \tag{3}$$

The second definition frequently referred to is the overall average heat transfer coefficient given by

$$\bar{h}_0 = \frac{\bar{q}_w}{\bar{T}_w - T_b} \tag{4}$$

that is, an average value based on the average heat flux and the difference between average wall temperature and fluid temperature. These two values, $\bar{h}_x$ and $\bar{h}_0$, are different (except in the special case when the flux and wall temperature are constant) and while this may be familar to the reader, the magnitude of the difference is perhaps less well known.

Referring again to Fig 1(b), the two average values are evaluated in the following way. At the interface let it be assumed that there is a uniform heat flux q_w and a linearly varying wall temperature, as a result of a particular set of conditions in the wall and in the fluid. For the temperature,

$$T_w = \bar{T}_w - m\left(\frac{L}{2} - x\right) \tag{5}$$

where

$$-\frac{2\bar{T}_w}{L} < m < \frac{2\bar{T}_w}{L}$$

which means that for any chosen value of the gradient m, with the mean temperature $\bar{T}_w$ constant, any linear distribution of T_w may be specified. The fluid temperature

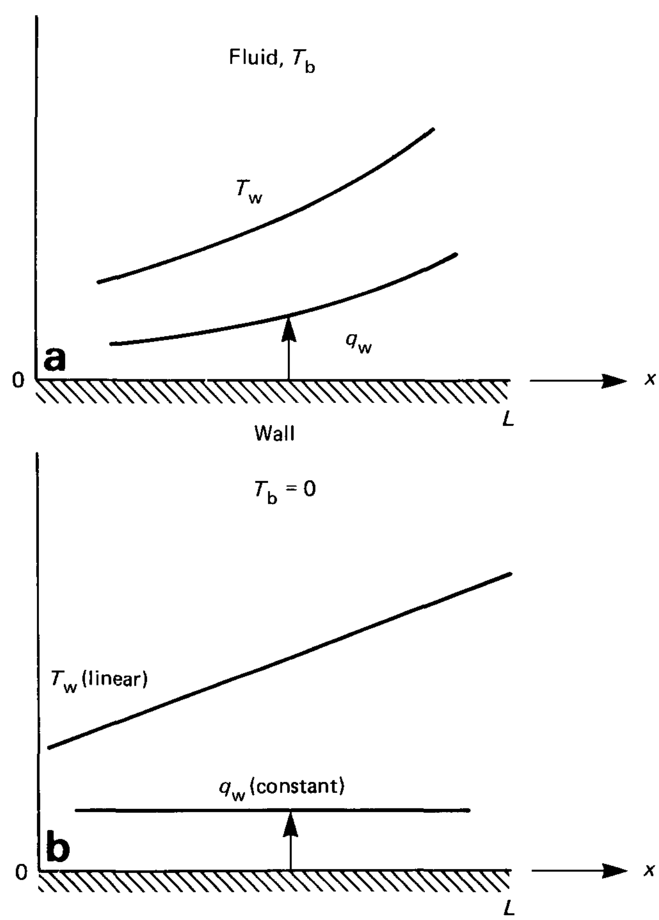

Fig 1 Wall heat transfer

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Table 1 Ratio of average heat transfer coefficients

m	$(\bar{h}_x/\bar{h}_0)$
0	1.0
0.5 $(\bar{T}_w/L)$	1.02
$(\bar{T}_w/L)$	1.10
1.5 $(\bar{T}_w/L)$	1.30

may be chosen to be zero. The average heat transfer coefficient $\bar{h}_x$ is now

$$\bar{h}_x=\frac{q_w}{L}\int_0^L\frac{dx}{T_w}=\frac{q_w}{L}\int_0^L\frac{1}{\bar{T}_w-m\left(\frac{L}{2}-x\right)}dx$$

whence,

$$\bar{h}_x=\frac{q_w}{Lm}\ln\left\{\frac{\bar{T}_w+\frac{mL}{2}}{\bar{T}_w-\frac{mL}{2}}\right\} \tag{6}$$

whereas, the overall average coefficient $\bar{h}_0$ is simply,

$$\bar{h}_0=\left(\frac{q_w}{\bar{T}_w}\right) \tag{7}$$

Of course, $\bar{h}_x=\bar{h}_0$ when $m=0$ and this may be verified by applying L'Hôpital's rule to Eq (6) for the limiting value $m=0$.

It now remains to evaluate Eq (6) and (7), and this is done by forming the ratio $(\bar{h}_x/\bar{h}_0)$, values of which are tabulated below for a range of values of the gradient m.

The results indicate that there may be a significant difference in the values of $\bar{h}_x$ and $\bar{h}_0$ (which depends on the gradient of the temperature along the surface in this case). $\bar{h}_x$ is greater than $\bar{h}_0$. The important point which emerges from this enquiry is that caution needs to be exercised when dealing with 'average heat transfer coefficient' and making comparisons between the various data. In experimental work, $\bar{h}_0$ appears to be the obvious choice in the calculation of the surface heat transfer, since $\bar{q}_w$ is obtainable directly from the power input data and $\bar{T}_w$ may be readily determined by averaging local wall temperature measurements. On the other hand, $\bar{h}_x$ is more appropriate in the general numerical prediction of heat transfer, since both local temperatures and their normal gradients at the wall are usually available in the computational procedure. Of course the difference between $\bar{h}_x$ and $\bar{h}_0$ is dependent on the particular heat transfer situation, those listed in Table 1 referring to the uniform wall flux and linear temperature gradient condition described earlier. (This thermal situation has been chosen specificially in order that the integration leading to Eq (6) may be effected analytically, since, otherwise, numerical techniques are involved.) However, it is very clear that in every case care is necessary when comparing data for average heat transfer coefficients, particularly when the sources of those data are of experimental and theoretical origin.